Start Where They Are: An Argument For Development-Centered Algebra Curriculum.

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Bruner (1960) proposes that American schools construct their math curriculum with the "aid of those with a high degree of vision and competence" (p. 19), namely mathematicians. He warns, "we can certainly ill afford as a nation to allow local inadequacies to inhibit the development of children" (Bruner, 1960, p. 11). Such high-minded ideals offer no guidance to classroom teachers, who society tasks with their application. Textbooks and standards also offer little instruction on how to bridge the developmental gap between professionally competent mathematicians and inequitably situated children. How should algebra teachers proceed? I will argue that algebra teachers who strive to reach Bruner's ideals and meet existing standards will succeed only by considering the developmental theories of Piaget and Vygotsky in designing and distributing their curricula. I will draw on recent literature from researchers who have looked specifically at the transition from arithmetic to algebra, and I will make reference to my own experience as an algebra teacher. Then, drawing on the contributions of Piaget and Vygotsky, I will build a case for algebra teachers to shift their focus to development-centered curriculum, starting with an assessment of each student's conceptual understanding of arithmetic and guiding their development of algebraic reasoning from there. I will also argue in favor of making curriculum decisions using pertinent knowledge of each student's socio-cultural background. If teachers do not *start where their students are*, both cognitively and socially, their students will learn very little from even the most cleverly crafted textbooks and standards.

American students entering an Algebra classroom often come from a plethora of schools and socio-cultural backgrounds. Drawing students from parochial, private, charter, and home schools, the Algebra classroom contains an amalgam of students with differing levels of preparation (e.g. number of years of mathematics education, time spent engaged in mathematical problem solving, and level of arithmetic mastery). The same students also bring to the classroom a diverse set of socio-cultural experiences with mathematics (e.g. the use of mathematics in their religions and beliefs, in their local communities, and in their homes), as well as varied levels of familial involvement and support (e.g. parental guidance, tutoring, time constraints, and technology access). Herscovics and Linchevsky (1994) suggest students entering algebra are also in the midst of grappling with “a cognitive gap” between arithmetic and algebra. Summarizing their findings using the vernacular of cognitive scientists (e.g. Anderson, 1995): the set of cognitive productions necessary to perform arithmetic operations are discontinuous with the set of productions required for algebraic processing. Many researchers observe that this discontinuity may result from the introduction of representative symbols, often in the form of letters (a, b, x, y, etc.) (e.g. Herscovics and Linchevsky, 1994, Goodson-Espy, 1998, Van Amerom, 2003, and Duval, 2006). The introduction of representative semiotics to students who lack the appropriate level of preparation and/or development to cognitively grasp them can lead to students’ failure to construct meaning for new representations and symbols, forcing them to perform meaningless operations without comprehension (Herscovics and Linchevsky, 1994). To avoid this result, teachers must *normalize* curriculum, accounting for the diversity of student preparation and support while simultaneously bridging the discontinuity students face in conceptualizing algebraic processes.

Meanwhile, standards and textbooks are designed to be of maximal application to a large numbers of students and teachers. They are therefore generalized ideals of mathematical knowledge and skill achievement. To use these general tools in the construction of normalized lesson plans, making them specifically relevant to a new set of students each year, the teacher must examine ways of assessing incoming students for existing cognitive productions as well as socio-cultural support structures. I will refer to this as *starting where they are*. Before discussing how teachers can implement such an approach, I will illustrate what can happen if they do not.

Piaget (1964) claimed, “A stimulus is a stimulus only to the extent that it is significant and it becomes significant only to the extent that there is a structure which permits its assimilation.” (p. 24) As I described above, if an algebra teacher attempts to start with the first topic in a textbook or the first objective in a set of standards, they risk teaching to students who do not have the appropriate cognitive productions (Piaget refers to them as structures) necessary for assimilation. According to Piaget this would render the teachings insignificant, leading students to ignore the teacher’s “stimulus.” In other words, if teachers attempt to cover material their students aren’t ready for, their lessons will not even be heard, let alone processed and understood. From my experience as an algebra teacher, this is referred to colloquially as “in one ear, out the other.” Teachers can choose to claim this is a result of a lack of motivation, attentiveness, or mathematical skill on the part of the student, but such reasoning empowers neither the teacher nor the student. Instead, by mapping stimulus to each student’s previously developed structures, teachers can begin to shape their curricular choices to make new lessons more accessible to their students.

Piaget’s assertion that students have a fundamental need to fit new information and experiences into the context of their existing cognitive structures implies students will either stop engaging in new algebraic concepts or misinterpret them in order to achieve cognitive homeostasis (Piaget, 1964). I have observed teachers referring to this behavior as “giving up” and “doing without understanding” respectively. Piaget (1964) also claims, “all development is composed of momentary conflicts and incompatibilities which must be overcome to reach a higher level of equilibrium” (p. 27). How many incompatibilities can a student struggle with before they give up in frustration or force a mathematically incorrect equilibrium? The answer will most likely depend on the student's existing socio-cultural support mechanisms. In order to engage in a process of continuously discovering cognitive conflicts and "overcoming" them, students must have the appropriate mechanisms of support to be successful (i.e. time, practice, peer/tutor guidance, and encouragement). Where students do not have sufficient access to such mechanisms, teachers must provide them. Failure to accurately assess each student's existing support mechanisms could lead teachers to believe that some students "try hard" while others just "give up." However, it is more functional to say that some students already possess the affordances necessary to achieve equilibration while others require them. This perspective empowers teachers to equitably design and distribute their curriculum, as long as they are willing to start where their students are.

I would now like to offer algebra teachers a couple strategies to begin practicing this approach. Vygotsky's Zone of Proximal Development (ZPD) affords teachers a model to use in constructing curricular linkages between where students are and where standards suggest they should be. The ZPD is bound by two developmental states, the student's current state (including their existing cognitive productions) and the maximum developmental state the student can demonstrate when assisted by a teacher or more experienced peer (Vygotsky, 1978). To establish each student's algebraic ZPD, the teacher would first determine the student's state of arithmetic comprehension, and then observe their performance on a series of guided exercises designed to challenge the student with increasingly advanced algebraic reasoning tasks. To determine each student's current cognitive understanding of arithmetic, Warren (2003) suggests teachers test student understanding of the associative law, the commutative law, and their facility with general processes of addition and division. She found that many students appear to be completing primary school without understanding these fundamental precursors to the cognitive productions of algebra (Warren 2003). By determining what topics need review before moving into the algebra curriculum, teachers can support future lessons by starting where their students are and reiterating fundamental concepts.

Once the teacher defines his or her students' base states, the teacher would then define their ZPDs through guided exercises, prompting students to abstract arithmetic operations as general algebraic processes. To construct these exercises, Warren suggests exploring "problem types that lead to general thinking and that lead to a countable number of answers." (Warren, 2003, p. 132) A teacher could, for example, ask students to solve a word problem for which no algebraic construction is necessary (where only arithmetic is required), then they could ask students what might happen to their solution if one of the needed pieces of information was unknown. Could they express their solution in terms of the unknown quantity? Could they create a symbol to represent the quantity when it is unknown? Once these tasks have been carried out, could they determine what limitations (if any) there might be on the unknown quantity and therefore the solution? Each of these tasks requires a different cognitive production. Some students will be able to perform one, two, or three of these tasks through the assistance of their peers or teacher. Students who cannot perform any of them, even with assistance, or who appear to struggle to understand the questions, may have smaller zones of proximal development and will require smaller initial training steps to develop. Those who quickly move forward through this guided exercise may not need as much support to begin working independently. By determining the ZPD of individual students, the algebra teacher can make equitable decisions on how to distribute time and attention. He or she can also begin customizing curriculum scaffolding to fit within students' ZPDs and start identifying which students can serve as peer tutors.

Conclusion

Algebra teachers face several challenges when helping students meet high standards. They must help students overcome gaps in understanding throughout the transition from arithmetic to algebra. They must draw lesson plans from textbooks that are written for mass consumption, while attempting to serve students with differing cultural backgrounds, levels of preparation, and existing support structures. Yet, in Bruner's words, "a skillful teacher can also experiment by attempting to teach what seems to be intuitively right for children… correcting as he goes" (Bruner, 1960, p. 53). The work of Piaget and Vygotsky offer algebra teachers a place to start experimenting and some suggestions on how to make corrections as they go. A development-centered curriculum prompts teachers to assess where students are coming from, what forms of support they have access to, and what cognitive steps they are capable of making under guidance. Armed with this information, teachers can make informed choices when constructing their curriculum to support the largest number of students equitably and effectively. The researchers, the textbook publishers, and the standards designers have made their contributions. Now it remains for algebra teachers to use these resources to design and distribute curriculum that starts where their students are and helps them develop to fruition.

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